



Pearson

# **Mark Scheme (Results)**

Summer 2017

Pearson Edexcel International A Level in  
Statistics S3 (WST03/01)

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## General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

# EDEXCEL IAL MATHEMATICS

## General Instructions for Marking

1. The total number of marks for the paper is 75.
2. The Edexcel Mathematics mark schemes use the following types of marks:
  - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
  - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
  - **B** marks are unconditional accuracy marks (independent of M marks)
  - Marks should not be subdivided.
3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
  - ft – follow through
  - the symbol  $\checkmark$  will be used for correct ft
  - cao – correct answer only
  - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
  - isw – ignore subsequent working
  - awrt – answers which round to
  - SC: special case
  - oe – or equivalent (and appropriate)
  - dep – dependent
  - indep – independent
  - dp decimal places
  - sf significant figures
  - \* The answer is printed on the paper
  - $\square$  The second mark is dependent on gaining the first mark
4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.
  5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
  6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
  7. Ignore wrong working or incorrect statements following a correct answer.

Question Number	Scheme									Marks	
1. (a)	Parrot	A	B	C	D	E	F	G	H	M1 M1 A1 dM1; A1 [5]	
	Rank Age	3	6	2	1	7	8	4	5		
	Rank Breeder	5	6	4	1	8	7	2	3		
	For finding the difference between each of the ranks and evaluating $\hat{a}d^2$										
	$\hat{a}d^2 = 4 + 0 + 4 + 0 + 1 + 1 + 4 + 4 = 18$										
	For use of the correct formula with their $\hat{a}d^2$										
$r_s = 1 - \frac{6(18)}{8(8^2 - 1)}; = 0.78571429...$									$\frac{11}{14}$ or awrt 0.786	A1	
(b)	$H_0: r = 0, H_1: r > 0$									Both hypotheses stated correctly	B1
	Critical Value = 0.8333 or CR: $r_s \geq 0.8333$									Critical value of 0.8333	B1
	Since $r_s = 0.7857...$ does not lie in the CR (or $0.7857... < 0.8333$ ), do not reject $H_0$									see notes	M1
	Either conclude that									A1ft	
	<ul style="list-style-type: none"> <li>the <u>breeder does not</u> have the ability to correctly <u>order parrots</u> by age, after examining them.</li> <li>there is <u>insufficient evidence</u> that the <u>breeder</u> can correctly <u>order parrots</u> by age.</li> </ul>										
									[4]	9	
<b>Notes</b>											
(a)	1 <sup>st</sup> M1	Attempt to rank for actual ages or breeder's estimates of ages. (At least 4 correct in either row - allow reverse rankings)									
	2 <sup>nd</sup> M1	Independent of 1 <sup>st</sup> M1 but these must be ranks.									
	3 <sup>rd</sup> dM1	<i>is dependent on 1<sup>st</sup> M1</i> for use of $1 - \frac{6(18)}{8(8^2 - 1)}$ with their $\hat{a}d^2$ .									
(b)	1 <sup>st</sup> B1	Both hypotheses correct in terms of $r$ or $r_s$ .									
	2 <sup>nd</sup> B1	Critical value of 0.8333									
	M1	For a correct statement relating their $r_s$ ( $ r_s  < 1$ ) with their c.v. where $ \text{their c.v.}  < 1$									
	A1ft	For a contextualised comment which is accepting $H_0$ , which must mention " <u>breeder</u> ", " <u>order</u> ", " <u>parrots</u> ", which conveys the idea that the breeder cannot order them correctly. All previous marks in part (b) must have been scored to award this one.									
	Note	Follow through their $r_s$ with 0.8333									
	Note	<b>Two-tailed test</b> Applying a two-tailed test scores a maximum of BOB1M1A0 <b>So Award SC B0B1</b> for $H_0: \rho = 0, H_1: \rho \neq 0$ followed by critical value $r_s = (\pm) 0.881$ and allow access to the M1 mark only.									

Question Number	Scheme		Marks																																	
2.	H <sub>0</sub> : There is no association between gender and (inspirational) message (independent) H <sub>1</sub> : There is an association between gender and (inspirational) message (dependent)		B1																																	
	<table border="1"> <thead> <tr> <th>Expected</th> <th>A</th> <th>B</th> <th>C</th> <th>Total</th> </tr> </thead> <tbody> <tr> <td>Male</td> <td>27.106...</td> <td>41.373...</td> <td>38.52</td> <td>107</td> </tr> <tr> <td>Female</td> <td>29.893...</td> <td>45.626...</td> <td>42.48</td> <td>118</td> </tr> <tr> <td>Total</td> <td>57</td> <td>87</td> <td>81</td> <td>225</td> </tr> </tbody> </table>		Expected	A	B	C	Total	Male	27.106...	41.373...	38.52	107	Female	29.893...	45.626...	42.48	118	Total	57	87	81	225	Some attempt at (Row Total)(Column Total) (Grand Total)	M1												
	Expected	A	B	C	Total																															
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				A1																																
	<table border="1"> <thead> <tr> <th>Observed</th> <th>Expected</th> <th><math>\frac{(O - E)^2}{E}</math></th> <th><math>\frac{O^2}{E}</math></th> </tr> </thead> <tbody> <tr> <td>25</td> <td>27.11</td> <td>0.1642...</td> <td>23.0542...</td> </tr> <tr> <td>37</td> <td>41.37</td> <td>0.4616...</td> <td>33.0916...</td> </tr> <tr> <td>45</td> <td>38.52</td> <td>1.0901...</td> <td>52.5701...</td> </tr> <tr> <td>32</td> <td>29.89</td> <td>0.1489...</td> <td>34.2589...</td> </tr> <tr> <td>50</td> <td>45.63</td> <td>0.4185...</td> <td>54.7885...</td> </tr> <tr> <td>36</td> <td>42.48</td> <td>0.9885...</td> <td>30.5085...</td> </tr> <tr> <td colspan="2">Totals</td> <td>3.2718...</td> <td>228.2718...</td> </tr> </tbody> </table>		Observed	Expected	$\frac{(O - E)^2}{E}$	$\frac{O^2}{E}$	25	27.11	0.1642...	23.0542...	37	41.37	0.4616...	33.0916...	45	38.52	1.0901...	52.5701...	32	29.89	0.1489...	34.2589...	50	45.63	0.4185...	54.7885...	36	42.48	0.9885...	30.5085...	Totals		3.2718...	228.2718...	At least 2 correct terms for $\frac{(O - E)^2}{E}$ or $\frac{O^2}{E}$ or correct expressions with their $E_i$ . Accept 2 sf accuracy for the dM1 mark.	dM1
	Observed	Expected	$\frac{(O - E)^2}{E}$	$\frac{O^2}{E}$																																
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$X^2 = \sum \frac{(O - E)^2}{E} \text{ or } \sum \frac{O^2}{E} - 225 ; = \text{awrt } 3.27$		$\sum \frac{(O - E)^2}{E} \text{ or } \sum \frac{O^2}{E} - 225$	dM1																																	
		awrt <b>3.27</b>	A1																																	
$n = (2 - 1)(3 - 1) = 2$		$n = 2$	B1																																	
$\chi^2_2(0.10) = 4.605 \Rightarrow \text{CR: } X^2 \geq 4.605$		<b>4.605</b>	B1ft																																	
[does not lie in the CR/not significant/Do not reject H <sub>0</sub> ]																																				
Either conclude that <ul style="list-style-type: none"> <li>there is <u>insufficient</u> evidence to support the <u>headteacher's</u> belief.</li> <li>there is <u>no association</u> between <u>gender</u> and <u>inspirational message</u>. (They are independent)</li> </ul>			A1																																	
			[10]																																	
			10																																	
<b>Notes</b>																																				
1 <sup>st</sup> B1	For both hypotheses. Must mention "gender" <b>and</b> "message" oe at least once. Use of "relationship" or "correlation" or "connection" or "link" is B0.																																			
1 <sup>st</sup> M1	Can be implied by at least one correct $E_i$ to 1 d.p.																																			
1 <sup>st</sup> A1	At least 5 expected frequencies correct awrt or trunc. 2 d.p. [No fractions]																																			
2 <sup>nd</sup> dM1	Dependent on 1 <sup>st</sup> M1 for at least 2 correct terms or correct expressions with their $E_i$																																			
2 <sup>nd</sup> A1	At least 5 correct terms to either 1 d.p. or awrt/trunc. 1.d.p. (may be implied by awrt 3.27)																																			
3 <sup>rd</sup> dM1	Dependent on 2 <sup>nd</sup> M1 For applying either $\sum \frac{(O - E)^2}{E}$ or $\sum \frac{O^2}{E} - 225$																																			
Note	If awrt 3.27 is seen (from a calculator) <b>without</b> the expected frequencies stated then award <b>special case</b> M0A0M1A1M1A1.																																			
2 <sup>nd</sup> B1	$n = 2$ . This mark can be implied by a correct critical value of 4.605																																			
3 <sup>rd</sup> B1ft	4.605 or ft their $\nu$																																			
4 <sup>th</sup> A1	Dependent on 3 <sup>rd</sup> M1 and 4.605. A correct contextualised conclusion which is accepting H <sub>0</sub> . Must mention either "headteacher's belief" or "gender" <b>and</b> "message".																																			
Note	Contradictory statements score A0. E.g. "significant, do not reject H <sub>0</sub> "																																			
Note	Condone "relationship" or "connection" here but <b>not</b> "correlation".																																			
Note	Hypotheses the wrong way round is A0																																			

Question Number	Scheme		Marks
3. (a)	$H_0 : m = 30 \quad H_1 : m \neq 30$		B1
	$z = \frac{28.2 - 30}{\frac{8.5}{\sqrt{75}}}; = -1.833936...$	$\pm \frac{28.2 - 30}{\frac{8.5}{\sqrt{75}}}$ or equivalent.	M1;
		awrt <u>-1.83</u>	A1
	Two tailed c.v.'s $Z = \pm 1.6449$ or CR: $Z \leq -1.6449$ or $Z \geq 1.6449$ or p-value = awrt 0.033 or awrt $0.034 < 0.05$ [in the CR/significant/Reject $H_0$ /"[0.033, 0.034]" < 0.05]		B1
	Conclude either <ul style="list-style-type: none"> <li>that the <u>mean age of gym customers is not 30 years.</u></li> <li>that the <u>manager's claim is not correct.</u></li> </ul>		A1
		[5]	
(b)	$\bar{X}$ is (approximately) <u>normally distributed</u>		B1
		[1]	
(c)	Assumed $s^2 = S^2$ or variance of sample = variance of population.		B1
		[1]	
		7	
<b>Notes</b>			
(a)	1 <sup>st</sup> B1	Both hypotheses correct.	
	M1	For standardising with 28.2, 30 and $\frac{8.5}{\sqrt{75}}$ (or awrt 0.981) [Allow use of $8.5 \times \sqrt{\frac{74}{75}}$ (=awrt 8.44)]	
	2 <sup>nd</sup> B1	Critical value of -1.6449 (compatible with sign of their test statistic) <b>or</b> a correct probability comparison.	
	2 <sup>nd</sup> A1	Dependent on M1 scored for a correct contextualised comment which is rejecting $H_0$ which is based on their z-value and their critical value with compatible signs, where $1.64 \leq  c.v.  \leq 1.65$ Contradictory statements score final A0. E.g. "significant, do not reject $H_0$ ".	
	<b>Alternative method for the "M1A1B1" marks:</b> Let $\bar{X}_c$ be the critical value of the sample mean.		
	$-1.6449 = \frac{\bar{X}_c - 30}{\frac{8.5}{\sqrt{75}}}$	M1: For $\frac{c - 30}{\frac{8.5}{\sqrt{75}}} = -1.6449 / -1.645 / -1.64 / -1.65$	
	So $\bar{X}_c = 28.38883812...$	A1: $\bar{X}_c =$ awrt 28.4	
		B1: Critical value of -1.6449	
Note	<b>One tailed test</b> SC: Applying a one-tailed test scores a maximum of B0M1A1B1A0 (Allow $\pm 1.2816$ to score the 2 <sup>nd</sup> B1)		
(b)	Allow in words e.g. " <b>sample mean</b> is normally distributed"		
(c)	B1	Also allow $s = S$ or standard deviation of sample = standard deviation of population.	

Question Number	Scheme		Marks																																																																					
4. (a)	$\hat{\lambda} = \frac{0(3) + 1(13) + 2(14) + 3(15) + 4(10) + 5(8) + 6(8) + 7(6) + 8(3)}{80} \left\{ = \frac{280}{80} \right\} = 3.5^*$		B1cso *																																																																					
			[1]																																																																					
(b)	$r = 80 \cdot \frac{e^{-3.5}(3.5)^3}{3!} \{ = 17.26283752... \}$ $\text{or } r = 80 \cdot (0.5366 - 0.3208) \{ = 17.264 \}$ $s = 80 - (2.42 + 8.46 + 14.80 + \text{their } r + 15.10 + 10.57 + 6.17 + 3.08) \{ = 2.14 \text{ or } 2.13716... \}$ $\text{or } s = 80 \cdot (1 - 0.9733) \{ = 2.136 \}$		M1																																																																					
	$r = 17.26$ (2dp), $s = 2.14$ (2dp)	At least one of either $r = \text{awrt } 17.26$ or $s = \text{awrt } 2.14$	A1																																																																					
		Both awrt $r = 17.26$ and awrt $s = 2.14$	A1																																																																					
			[3]																																																																					
(c)	<p><math>H_0</math>: Poisson distribution is a suitable model.  <math>H_1</math>: Poisson distribution is not a suitable model.</p>		B1																																																																					
	<table border="1"> <thead> <tr> <th># calls</th> <th><math>O_i</math></th> <th><math>E_i</math></th> <th>Comb <math>O_i</math></th> <th>Comb <math>E_i</math></th> <th><math>\frac{(O_i - E_i)^2}{E_i}</math></th> <th><math>\frac{O_i^2}{E_i}</math></th> </tr> </thead> <tbody> <tr> <td>0</td> <td>3</td> <td>2.42</td> <td rowspan="2">16</td> <td rowspan="2">10.88</td> <td rowspan="2">2.4094...</td> <td rowspan="2">23.5294...</td> </tr> <tr> <td>1</td> <td>13</td> <td>8.46</td> </tr> <tr> <td>2</td> <td>14</td> <td>14.80</td> <td>14</td> <td>14.80</td> <td>0.0432...</td> <td>13.2432...</td> </tr> <tr> <td>3</td> <td>15</td> <td>17.26</td> <td>15</td> <td>17.26</td> <td>0.2959...</td> <td>13.0359...</td> </tr> <tr> <td>4</td> <td>10</td> <td>15.10</td> <td>10</td> <td>15.10</td> <td>1.7225...</td> <td>6.6225...</td> </tr> <tr> <td>5</td> <td>8</td> <td>10.57</td> <td>8</td> <td>10.57</td> <td>0.6249...</td> <td>6.0549...</td> </tr> <tr> <td>6</td> <td>8</td> <td>6.17</td> <td>8</td> <td>6.17</td> <td>0.5428...</td> <td>10.3728...</td> </tr> <tr> <td>7</td> <td>6</td> <td>3.08</td> <td rowspan="2">9</td> <td rowspan="2">5.22</td> <td rowspan="2">2.7372...</td> <td rowspan="2">15.5172...</td> </tr> <tr> <td><math>\geq 8</math></td> <td>3</td> <td>2.14</td> </tr> <tr> <td colspan="5" style="text-align: center;">Totals</td> <td>8.3759...</td> <td>88.3759...</td> </tr> </tbody> </table>		# calls	$O_i$	$E_i$	Comb $O_i$	Comb $E_i$	$\frac{(O_i - E_i)^2}{E_i}$	$\frac{O_i^2}{E_i}$	0	3	2.42	16	10.88	2.4094...	23.5294...	1	13	8.46	2	14	14.80	14	14.80	0.0432...	13.2432...	3	15	17.26	15	17.26	0.2959...	13.0359...	4	10	15.10	10	15.10	1.7225...	6.6225...	5	8	10.57	8	10.57	0.6249...	6.0549...	6	8	6.17	8	6.17	0.5428...	10.3728...	7	6	3.08	9	5.22	2.7372...	15.5172...	$\geq 8$	3	2.14	Totals					8.3759...	88.3759...	M1  M1
# calls	$O_i$	$E_i$	Comb $O_i$	Comb $E_i$	$\frac{(O_i - E_i)^2}{E_i}$	$\frac{O_i^2}{E_i}$																																																																		
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	$n = 7 - 1 - 1 = 5$			B1ft																																																																				
	$\chi^2_5(0.05) = 11.070 \Rightarrow \text{CR: } X^2 \geq 11.070$			B1ft																																																																				
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<b>Notes</b>																																																																								
(a)	B1cso*	At least 2 non-zero products shown and divide by 80 to achieve 3.5*																																																																						
(c)	1 <sup>st</sup> B1	Must have both hypotheses and mention Poisson at least once. Inclusion of 3.5 for / in is 1 <sup>st</sup> B0																																																																						
	1 <sup>st</sup> M1	For a correct method of pooling the classes at both ends [ft their $s$ value]																																																																						
	2 <sup>nd</sup> M1	For an attempt at the test statistic, at least 3 correct expressions/values (to awrt/truncated 2 d.p.)																																																																						
	1 <sup>st</sup> A1	awrt 8.38 or awrt 8.39 (This implies the both M1 marks)																																																																						
	2 <sup>nd</sup> B1ft	For their evaluated $n - 1 - 1$ . i.e. realising that they must subtract 2 from their $n$ .																																																																						
	3 <sup>rd</sup> B1ft	Correct ft for their $\chi^2_k(0.05)$ , where $k = n - 1 - 1$ from their $n$ . (May see 9.488, 12.592, 14.067)																																																																						
	2 <sup>nd</sup> A1	Dep. on at least 1 M1 mark for a correct conclusion which is accepting $H_0$ .																																																																						
	<b>Note</b>	No follow through on their hypotheses if they are stated the wrong way round.																																																																						
	<b>Note</b>	Contradictory statements score A0. E.g. "significant, do not reject $H_0$ "																																																																						
	<b>Note</b>	Condone the mention of Po(3.5) in conclusion.																																																																						

Question Number	Scheme	Marks
5. (a)	Label beginners 1 – 452, intermediates 1 – 251, professionals 1 – 97 Use <u>random numbers</u> to select a ... Simple random sample of <u>28 beginners</u> , <u>16 intermediates</u> and <u>6 professionals</u> .	M1 M1 A1 <b>[3]</b>
(b)	<b>Any one of</b> <ul style="list-style-type: none"> <li>Enables estimation of statistics/sampling errors for each strata.</li> <li>Reduces variability.</li> <li>More representative of the population/reflects population structure</li> </ul>	B1 <b>[1]</b>
(c)	$H_0 : m_I - m_B = 3$ $H_1 : m_I - m_B > 3$	B1; B1
	$s.e. = \sqrt{\frac{38.1}{60} + \frac{57.3}{80}} \{ = 1.162432794... \}$	M1
	$z = \frac{36.9 - 31.7 - 3}{"1.1624..."} ; = 1.89258...$	dM1; awrt <u>1.89</u> A1
	One tailed c.v. $Z = 1.6449$ or CR : $Z \geq 1.6449$ or p-value = awrt $0.029 < 0.05$ [in the CR/significant/Reject $H_0$ /"0.029" < 0.05]	B1
	Conclude either that the <ul style="list-style-type: none"> <li><u>mean score of intermediates</u> is more than <u>3 greater</u> than the <u>mean score of beginners</u>. (oe)</li> <li><u>manager's belief is correct</u>.</li> </ul>	A1
		<b>[7]</b>
		<b>11</b>
	<b>Alternative method for "2<sup>nd</sup> M1, 1<sup>st</sup> A1, 3<sup>rd</sup> B1" marks:</b> Let $D = \bar{x}_I - \bar{x}_B$	
	$1.6449 = \frac{D - 3}{1.1624...}$	<b>dependent upon the 1<sup>st</sup> M1 for</b> dM1:
	So, $D = 4.912...$	$\frac{D - 3}{\text{their "1.1624..."}} = 1.6449 / 1.645 / 1.64 / 1.65$ $D = \text{awrt } 4.91 \text{ and } D_{\text{obs}} = 5.2$ A1
	$D_{\text{obs}} = 36.9 - 31.7 = 5.2$	[1.64, 1.65] B1

Notes		
(a)	1 <sup>st</sup> M1 2 <sup>nd</sup> M1 A1	for a suitable numbered/labelled list for each ability level for use of random numbers/sample to select beginners, intermediates and professionals. <b>(dependent on either the 1<sup>st</sup> or the 2<sup>nd</sup> M1 mark)</b> For <u>28 beginners</u> , <u>16 intermediates</u> and <u>6 professionals</u> .
(c)	1 <sup>st</sup> B1 2 <sup>nd</sup> B1 <b>Note</b>	$H_0 : m_I - m_B = 3$ oe $H_1 : m_I - m_B > 3$ oe If $m_1, m_2$ used then it must be clear which one refers to intermediates/beginners.
	1 <sup>st</sup> M1	$s.e. = \sqrt{\frac{38.1}{60} + \frac{57.3}{80}}$ . May be implied by s.e. = awrt 1.16
	2 <sup>nd</sup> dM1 1 <sup>st</sup> A1 3 <sup>rd</sup> B1 2 <sup>nd</sup> A1	Condone minor slips e.g. $\sqrt{\frac{38.1}{80} + \frac{57.3}{60}}$ Dependent upon the 1 <sup>st</sup> M1. (follow through their s.e. if 1 <sup>st</sup> M1 mark has been awarded) awrt 1.89 $1.64 \leq  C.V.  \leq 1.65$ (compatible sign with their test statistic) or a correct probability comparison. Dep. on all M1 and B1 marks scored for contextualised comment which is rejecting $H_0$ .

Question Number	Scheme		Marks	
6. (a)	$\bar{x} = 230.5$ ; 95% confidence limits for $m$ are			
	$230.5 \pm 1.96 \cdot \frac{1.2}{\sqrt{5}}$	their $\bar{x} \pm z \cdot \frac{1.2}{\sqrt{5}}$	M1	
			$z = 1.96$	B1
	= (229.44815..., 231.55185...) = awrt(229.4, 231.6)		At least one end-point is correct.	A1
			Both end-points are correct.	A1
			<b>[4]</b>	
	(b)	{ Let $X$ = number of confidence intervals that <i>don't contain</i> $m$ }		
{ So $X \sim$ } B(20,0.05)			M1	
{ $P(X > 3)$ } = $1 - P(X \leq 3)$ or $1 - 0.9841$			A1	
= 0.0159		awrt <b>0.0159</b>	A1	
		<b>[3]</b>		
		<b>7</b>		
<b>Notes</b>				
(b)	<b>M1</b> <b>1<sup>st</sup> A1</b> <b>2<sup>nd</sup> A1</b>	Writing or using either $X \sim B(20,0.05)$ or $Y \sim B(20,0.95)$ $1 - P(X \leq 3)$ or $1 - 0.9841$ or $P(Y \leq 16)$ . Can be implied by the final answer. awrt 0.0159		

Question Number	Scheme		Marks	
7. (a)	$A = \frac{X_1 + X_2 + X_3 + Y_1 + Y_2}{5}$ , $X \sim N(30, 4.5^2)$ , $Y \sim N(20, 3.5^2)$ ; $X, Y$ are independent.			
	$E(A) = \frac{3(30) + 2(20)}{5}$ or $\text{Var}(A) = \frac{3(4.5)^2 + 2(3.5)^2}{25}$	A correct method for finding $E(A)$ or $\text{Var}(A)$	M1	
	$E(A) = 26$ or $\text{Var}(A) = 3.41$	At least one of either $E(A) = 26$ or $\text{Var}(A) = 3.41$	A1	
		Both $E(A) = 26$ and $\text{Var}(A) = 3.41$	A1	
	{So $A \sim N(26, 3.41)$ }			
	$\{P(A < 24) = \}$ $P\left(Z < \frac{24 - 26}{\sqrt{3.41}}\right)$	Standardising ( $\pm$ ) with their mean and their standard deviation	M1	
	$= P(Z < -1.08306\dots)$			
	$= 1 - 0.8599$		M1	
	$= 0.1401$ (or 0.139391...)	<b>0.14</b> or awrt <b>0.140</b> or awrt <b>0.139</b>	A1	
	<b>[6]</b>			
	(b)	$W \sim N(m, 2.8^2)$ ; $P(W - X < 4) = 0.1$ $W, X$ are independent.		
		$\{E(W - X) = E(W) - E(X) = m - 30\} \supset E(W - X) = m - 30$	$E(W - X) = m - 30$	B1
$\{\text{Var}(W - X) = \} 2.8^2 + 4.5^2 \{= 28.09\}$		$2.8^2 + 4.5^2$	M1	
{So $W - X \sim N(m - 30, 28.09)$ }				
$\{P(W - X < 4) = 0.1\} \Rightarrow P\left(Z < \frac{4 - (m - 30)}{\sqrt{2.8^2 + 4.5^2}}\right) = 0.1$				
$\frac{4 - (m - 30)}{\sqrt{2.8^2 + 4.5^2}} = k$ ( $= -1.2816$ )		Standardising ( $\pm$ ) with their mean which is in terms of $m$ and their standard deviation and setting the result equal to $k$ , where $ k $ is in the interval $[1.28, 1.29]$ .	M1	
		$\pm 1.2816$ or awrt $\pm 1.2816$	B1	
		Correct equation . See notes	A1	
$\{m = 34 + 1.2816(5.3) \supset\} m = 40.792\dots (= 40.784 \text{ from using } -1.28)$	awrt <b>40.8</b>	A1		
<b>[6]</b>				
<b>Notes</b>				
(a)	3 <sup>rd</sup> M1	For a probability tail compatible with 24 and their mean		
(b)	2 <sup>nd</sup> M1	Allow $\pm \frac{4 - \text{their } E(W - X)}{\sqrt{\text{their } \text{Var}(W - X)}} = k$ , where $ k $ is in the interval $[1.28, 1.29]$		
	2 <sup>nd</sup> B1	For either $-1.2816$ or $1.2816$		
	1 <sup>st</sup> A1	E.g. Allow $\frac{4 - (m - 30)}{\sqrt{2.8^2 + 4.5^2}} = [-1.29, -1.28]$ or $\frac{(m - 30) - 4}{\sqrt{2.8^2 + 4.5^2}} = [1.28, 1.29]$		

Question Number	Scheme	Marks
<b>8.</b>	$X$ follows a continuous uniform distribution over $[a + 3, 2a + 9]$ ; $Y = \frac{2\bar{X}}{3} + k$	
(a)	$\{E(\bar{X}) = m\} = \frac{2a + 9 + a + 3}{2}$	M1
	$= \frac{3a}{2} + 6$ or $\frac{3a + 12}{2} \neq a$ . {So $\bar{X}$ is a biased estimator.}	A1
		[2]
(b)	bias $\left\{ = \frac{3a}{2} + 6 - a \right\} = \frac{1}{2}a + 6$ or $\frac{a + 12}{2}$ (allow $\pm$ )	B1ft
		[1]
(c)	$\left\{ E(Y) = \frac{2}{3}E(\bar{X}) + k = a \Rightarrow \right\} \frac{2}{3}\left(\frac{3a}{2} + 6\right) + k = a$	M1
	$\{a + 4 + k = a \Rightarrow\} k = -4$	$k = -4$ A1
		[2]
(d)	$\left\{ \hat{a} = \frac{2}{3}\bar{X} - 4 \Rightarrow \right\} \hat{a} = \frac{2}{3}(7.8) - 4 \{= 1.2\}$	M1
	Max value = $2(1.2) + 9$	M1
	$= 11.4$	11.4 or $11\frac{2}{5}$ or $\frac{57}{5}$ A1
		[3]
		8

**Notes**

(a)	M1	Using the formula $\left(\frac{b+a}{2}\right)$ or obtaining $\frac{3a+12}{2}$ or $\frac{3a}{2} + 6$
	A1	$\frac{3a}{2} + 6$ or $\frac{3a+12}{2}$ <b>and</b> $\neq a$ .
(b)	B1ft	bias = $\pm\left(\frac{1}{2}a + 6\right)$ or $\pm\left(\frac{a+12}{2}\right)$ or ft their $\mu$ provided $\mu \neq a$
(c)	M1	Sets $\frac{2}{3}(\text{their } E(\bar{X})) + k = a$ . This mark can be implied.
	A1	$k = -4$ . <b>Note</b> that $k = -4$ with no working is M1 (implied) A1.
(d)	1 <sup>st</sup> M1	An attempt to use the sample data given to find $\frac{2}{3}\bar{x} + \text{"their } k\text{"}$ .
		Allow full expression for $\bar{x}$ or $\frac{\sum x}{n}$ . ( <b>Note</b> that from the data $\bar{x} = 7.8$ )
	2 <sup>nd</sup> M1	2 <sup>nd</sup> "their $a$ " + 9 where their $a$ is a function of the sample mean – which has been found by applying $\frac{\sum x}{n}$ from the data values given in the question.
	A1	11.4 cao
	<b>Note</b>	$2(10.6) + 9 = 30.2$ is M0M0A0

